Optimization of Organ Conductivity for the Forward Problem of Electrocardiography

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Introduction

- The forward problem of electrocardiography defines the relationship between epicardial and body surface potentials that is fundamental to non-invasive mapping. Forward models incorporating inhomogeneous structures are more accurate than homogeneous [1,2] though a difference between the forward and recorded potentials remains.
- Theoretically, given a simultaneous measures of epicardial and body surface potentials, an optimal forward model could be found by optimizing the conductivities within the model. Identification of an optimized transfer matrix would provide a better forward model than both a uniform isotropic model and a more complex physiologically based model.
- This study examines a method for optimizing the conductivities within a torso model using an in-vivo experimental data set [1].

Optimization Procedure

The torso domain is denoted by Ω, covering the volume between the epicardium and the body surface

$$\Omega = \cup_{i\in\{f,m,l,c\}}\Omega_i,$$

where $\Omega_{f,m,l,c}$ are the fat, skeletal muscle, lung and cavity volumes, their respective conductivities denoted by $\sigma_{f,m,l,c}$. The boundary of the torso domain is defined by $\partial\Omega = \Sigma \cup \Gamma_{ext}$, where Σ represents the epicardial surface and Γ_{ext} the external boundary of the body surface. We denote the torso potential in Ω by u_T. The test data provides the electrical potentials on both Σ and Γ_{ext}. Moreover, we know that the current flux over the body surface is equal to zero. In order to estimate the values of each organ conductivity, we construct the following quantity of interest:

$$\begin{cases} I(\sigma_f,\sigma_m,\sigma_l,\sigma_c) = \frac{1}{2}\|u_T - d\|^2_{L^2(\Gamma_{ext})}, \\ \quad with\ u_T\ solution\ of: \\ \begin{cases} div(\sigma_T\nabla u_T) = 0, in\ \Omega, \\ u_T = h, on\ \Sigma, \\ \sigma\nabla u_T\cdot \boldsymbol{n}_T = 0, on\ \Gamma_{ext} \end{cases} \end{cases}$$

Using the fmincon function of Matlab 2013b, the cost function, $I(\sigma_{f,m,l,c})$, is minimized and the optimal conductivites for all four organs found. The derivative of I over σ_i was computed initially using the default finite difference approache ($Grad_{FD}$). Comparison were then made with results computing the derivative directly, that is:

$$\frac{\partial I}{\partial\sigma_i}(\sigma_f,\sigma_m,\sigma_l,\sigma_c) = \int_{\Gamma_{ext}}\frac{\partial u_T}{\partial\sigma_i}(u_T - d)$$

As the derivative of u_T cannot be computed directly, an adjoint method was used. That is $H^1(\Omega)$ is denoted by the set of functions $\emptyset\colon \Omega\to\mathbb{R}$, such that $\int_\Omega \emptyset^2 < \infty$ and $\int_\Omega |\nabla\emptyset|^2 < \infty$. $H^1_\Sigma(\Omega)$ is denoted by the set of functions $\psi \in H^1(\Omega)$ such that $\psi/_\Sigma = 0$. For every $(\sigma_f,\sigma_m,\sigma_l,\sigma_c,u,\lambda) \in H^1(\Omega)\times H^1_\Sigma(\Omega)$, we define a Lagrangian function as

$$L(\sigma_f,\sigma_m,\sigma_l,\sigma_c,u,\lambda) = \frac{1}{2}\|u_T - d\|^2_{L^2(\Gamma_{ext})} + \int_\Omega \sigma_T\nabla u\nabla\lambda$$

Thus , $I(\sigma_{f,m,l,c}) = L(\sigma_{f,m,l,c}, u, \lambda)$ for every $\lambda\in H^1_\Sigma(\Omega)$. The gradient of I with respect to σ_i is given by

$$\begin{cases} \frac{\partial I(\sigma_f,\sigma_m,\sigma_l,\sigma_c)}{\partial\sigma_i} = \int_{\Omega_i}\nabla u_T\nabla\lambda, \\ \quad with\ \lambda\ solution\ of: \\ \begin{cases} div(\sigma_T\nabla\lambda) = 0, in\ \Omega, \\ \lambda = 0, on\ \Sigma, \\ \sigma\nabla\lambda\cdot\boldsymbol{n}_T = -(u_T - d), on\ \Gamma_{ext} \end{cases} \end{cases}$$

This method allows us to obtain the derivative of the objective function over the four conductivity parameters, by only solving two Laplace equations: The first is the state equation to obtain u_T and the second is the adjoint equation to obtain λ. The derivative is then obtained by integrating the scalar product of the gradients of u_T and λ over each of the four organ domains.

Methods

- **Database**
  - Epicardial potentials were recorded in-vivo using an elastic sock (239 electrodes), in an anesthetized, closed-chest pig [1].
  - Post-mortem MRI was used to create a torso model (Fig 1), and epicardial electrode locations were captured with a multi-axis digitizing arm.
  - « Gold standard » torso potentials (d) were computed at 180 points from measured epicardial potentials using a finite-element defined forward model (Fig 1), and $\sigma_f = 0.04$, $\sigma_m = 0.40$, $\sigma_l = 0.05$, $\sigma_c = 0.22$ $mSmm^{-1}$.

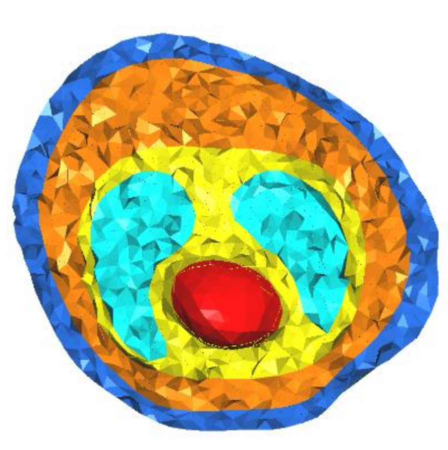

Fig 1. Cranial view of finite element torso model.

- **Optimization and Analysis**
  - Initial conductivities were defined using a Monte Carlo simulation, using four values for each conductivity from ±50% the max and min conductivities from the literature.
  - The sensitivity of the optimization procedure was tested by varying levels of signal noise (SN) on d, and torso electrode localization error (LE), for six different time points spanning ventricular depolarization.
  - To determine differences, a paired t-test was used for normally distributed data, and a two-sided Wilcoxon signed rank test for non-normal. Statistical significance was accepted for $p < 0.05$.

Sensitivity Analysis

- Given the correct fat conductivity, Monte Carlo simulations yielded initial conductivities of $\sigma_l = 0.06$, $\sigma_m = 0.35$, $\sigma_c = 0.13$ $mSmm^{-1}$, for all levels of signals noise, vest error, and for all time steps.
- In the following analysis, $Grad_{FD}$ was used to define the gradient during optimization
- **Signal Noise (SN)**
  - Gaussian SN was added to d, and conductivities optimized.
  - SN was created using a random nulber generator wth a standard deviation (SD) from 2 to 512 uV.
  - All three conductivities were accurately estimated (RE < 10 %) for up to 0,20 mv (Fig 2)
- **Vest Electrode Localization Error (LE)**
  - Gaussian error was added to each vest electrode and conductivities optimized.
  - The direction of LE was defined for each electrode by picking a random point on the surface of a unit sphere, with the distance defined with a mean from 0,02 to 2,56
  - All conductivities were accurately estimated (RE < 10 %) for all levels of LE (Fig 2)
- **Combined Error**
  - Conductivities were optimized combining a SN of 0.05 mV and a LE 9 mm (typical signal noise and geometric error levels)
  - All conductivities were accurately estimated for all signals (Fig 4)

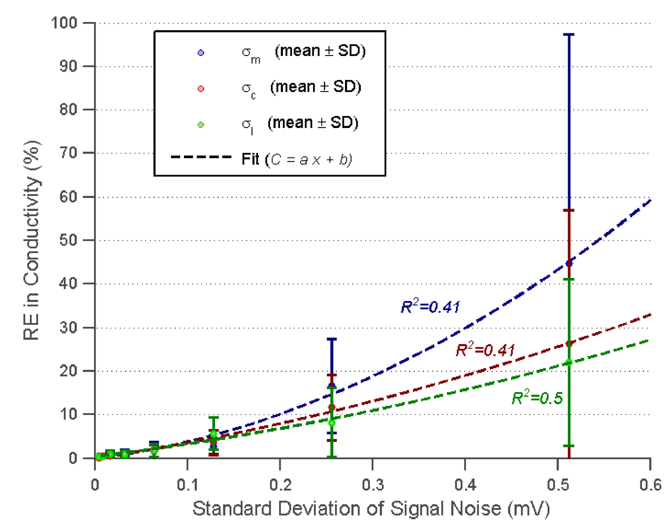

Fig 2. Relative Error (RE) in conductivity over different levels of SN

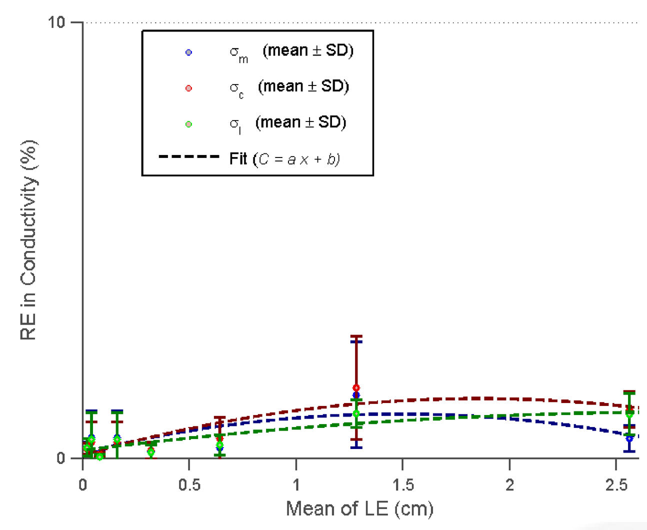

Fig 3. Relative Error (RE) in conductivity over different levels of LE

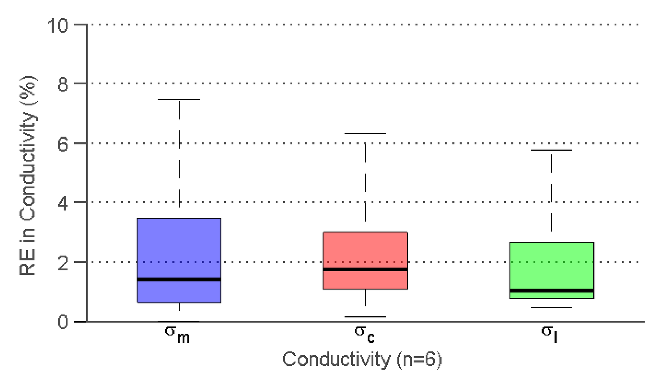

Fig 4. Relative Error (RE) in conductivity when combining 0.05 mV SN and 0.9 cm LE

Gradient Method

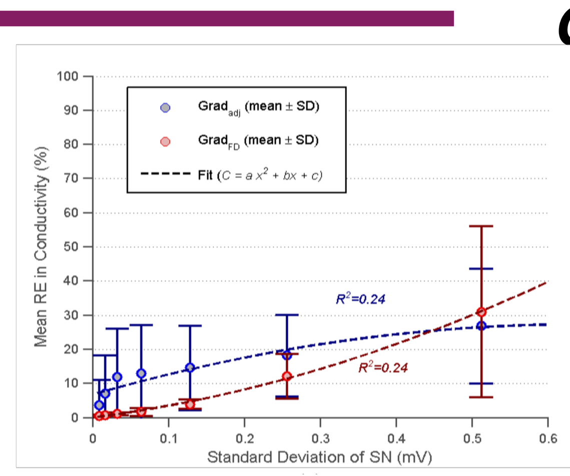

Fig 5. Mean RE in conductivity over SN for $Grad_{FD}$ and $Grad_{adj}$

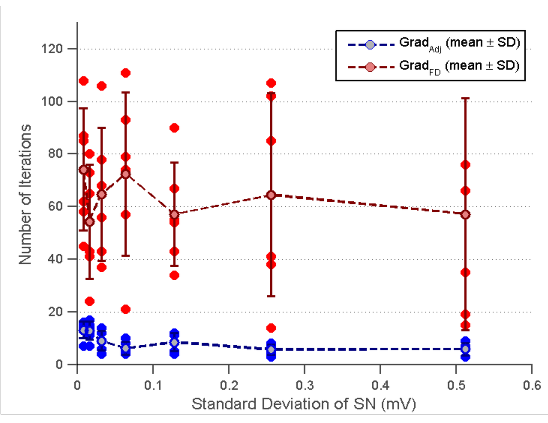

Fig 7. The number of iterations for computation over SN comparing $Grad_{FD}$ and $Grad_{adj}$

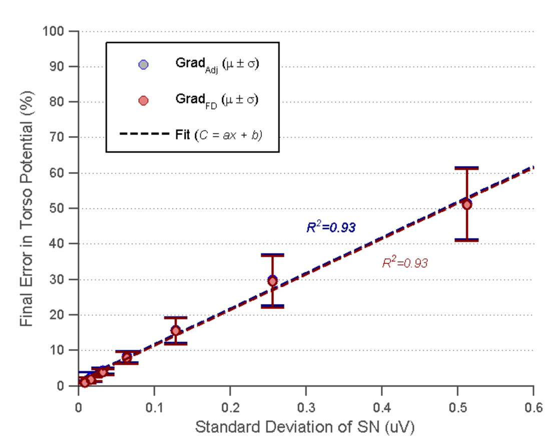

Fig 6. Final RE in torso potentials over SN for $Grad_{FD}$ and $Grad_{adj}$

- The accuracy and speed of optimization using $Grad_{adj}$ were compared to using $Grad_{FD}$ over different levels of SN
- Overall, $Grad_{adj}$ resulted in a significantly larger average error in conductivity ($p < 0.0043$) than $Grad_{FD}$. (Fig 5).
- $Grad_{adj}$ is substantially more computationally efficient than GradFD, requiring 45 to 64 fewer iterations to converge to a solution for all levels of SN (Fig 6)
- The final cost function values obtained with $Grad_{adj}$ were not substantially different from those using $Grad_{FD}$ (Fig 7).
- The gradient calculated using $Grad_{adj}$ likely resulted in small step sizes, and optimization was exited due to a step size or a change in cost function below threshold.

Conclusions

- Given experimental data with simultaneous epicardial and torso recordings, conductivities within the torso can be accurately computed under typical signal noise and geometric error levels.
- Though generally more robust when you include directly computed gradients, $Grad_{adj}$ was less robust to SN, though more computationally efficient than the standard $Grad_{FD}$. This may be due to the optimization process was exiing due to a step size or a change in cost function below threshold with $Grad_{adj}$.

Reference

[1] Bear LR, Cheng LK, LeGrice IJ, Sands GB, Lever NA, Paterson DJ, et al. Forward Problem of Electrocardiography. Circ Arrhythmia Electrophysiol. 2015;8(3):677–84.
[2] Stanley PC, Pilkington TC, Morrow MN, Ideker RE. An assessment of variable thickness and fiber orientation of the skeletal muscle layer on electrocardiographic calculations. IEEE Trans Biomed Eng. 1991 Nov;38(11):1069–76.